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ADAPTIVE MOTION SYNTHESIS BY QUALITATIVE APPROACH

Fangde Liu. *Mechanical Engineering Department, Imperial College London.*

Xiaosong Yang. *National Center for Computer Animation, Bournemouth University.*

Jianjun Zhang. *National Center for Computer Animation, Bournemouth University.*

ABSTRACT

Physics-based motion synthesis is usually difficult and takes long computational time. However biological research works show that human beings and animals take very little effort to control their motion. The idea is that instead of controlling motion in every detail, natural animals may only maintain or tweaks some qualitative properties of its motion and utilizes the complicate interaction between body and environment. Inspired by this theory, in this paper, a novel method is proposed for motion synthesis. Based on the theory of qualitative dynamics, adaptive motion control is achieved through manipulating the topological structure of the dynamic system, which enhance the structural stability, rather than counteracting the perturbation effects. Compared with current methods, the new method is extremely efficient for it requires little computation and could be accelerated by GPU.

KEYWORDS

Physics Based Motion Synthesis, Qualitative Dynamics, Character Motion Synthesis.

1. INTRODUCTION

Character Motion Synthesis (CMS) research aims at generating motion for virtual characters. It is a topic of significant value in terms of theory and application. The challenge of CMS is not to make characters move, but to make them lifelike. Underlying this challenge is the marvelous human ability of motion perception. In real life, people's motion is very similar, yet individuals vary considerably. From the varieties in motion details, humans can infer mental states, health conditions or the surrounding environment.

Nowadays in industry, high quality motions are mainly generated manually. However, because the virtual characters are complex and contain a large number of joints, believable animations requires lots of tedious work. To make it worse, reusing motion animation is also difficult and prone to artifacts. Therefore high level animation tools are badly needed.

To save animator from these tedious manual works, many researchers are trying to generate lifelike motions automatically by simulating the dynamics of body, environment and the neural control system. However since each virtual character is full of redundant degree of freedom, it not only increases the computational load, but also makes the solution nondeterministic.

Although animals have fascinated us for thousands of years, we still do not fully understand how they move. Animals are very different from artificial machines and such comparisons may reflect the biological motor control principle.

Degrees of freedom (DOFs): From a mechanical perspective, animals have many more DOFs than their artificial counterparts. An artificial ship can be approximated by a simple rigid body; whereas the flexible spine of a fish is made up of tens of DOFs. In principle, the extra DOFs allows for more variations in adapting the environment. However, for the control system, too many extra DOFs become a disaster because of the extra computational burden. For a human to take one step, the neural system controls more than 600 muscles. Even with nowadays computer, solving this dynamics directly would spend thousands of hours.

Versatility: Most artificial machines are designed with a single purpose, while animals are capable of unlimited tasks. Many biological functions which are often neglected by CMS research, such as feeding, breeding, language and vision, depend on motor control. Besides walking, swimming and many other styles of locomotion, we utilize many tools, such as cars, skates, bicycles and tennis rackets. Following traditional control methods, it seems that unlimited resources need to be allocated for motor control, while biological research shows motor control requires very few mental resources.

Performance: Although the problem of biological motor control is more complex, the resulting performance surpasses artificial machines in many aspects. Natural motions are more **Robust:** A human can maintain walking stability on rough terrains which would be inaccessible for vehicles. **Manoeuvrability and speed:** Typical modern airplanes travel at a maximum of 32 body length/sec and yaw at 720 deg/sec. While pigeons may travel at 75 body length/sec, yaw at about 5000 deg/sec. **Energy Efficiency:** The energy consumed by a walking human is only 5% of that for the world famous humanoid ASIMO.

In this paper, we introduce the Qualitative Control Theory (QCT) to tackle motion synthesis problem. The hypothesis is that natural motion control is based on the property of a structural stable autonomous system, and the key factor of motor control is the topological structure of the dynamic motion systems. So in our method, only the qualitative properties of motions are controlled. In this framework, adaptation to different environment or changing of character conditions will be produced automatically with very little control effort. All the above three natural motion features can be achieved from our system. Our method works especial efficient for repetitive and low energy motion tasks which are most challenging for motion synthesize. Compared with other methods, our approach is more computational efficient and has the potential to be further accelerated by GPU.

2. RELATED WORK

2.1 Dynamic Motion Synthesis and Control

Dynamic Motion Synthesis tries to synthesize character motion through simulation the dynamics of the mechanic structure of character body which is usually modeled as a linked rigid body system [Baraff, 1994, Mirtich, 1996, Stewart and Trinkle, 2000]. The generated motion are normally physical feasible. However, for character motion synthesis, the most difficult task is to design an efficient control method that mimics functionality of a real biological neural system.

Some early research applied classical control methods like PD controller [Raibert andHodgins, 1991] for locomotion. Later research [Hodgins et al., 1995] applied the same method for different tasks like running, bicycling, vaulting and balancing. Limit Circle Control (LCC) [Laszlo et al., 1996] provides an alternative method for lower energy locomotion animation. However both the classical PD controller and Limit Circle Controller track predefined motion trajectories and eliminated perturbations, thus both of them are not good for adaptive motions.

Because of the redundant DOFs, in most cases, motion solutions are not unique. Many optimization methods have been applied to choose the “best” motion. For dynamic methods, a reasonable choice is to minimize the energy cost V, such that $V = \int_{t_0}^{t_1} F_a(x)^2\, dt$. Where F_a a is the active force generated by actuators like motors or muscles. This is introduced to CMS research as the influential **Spacetime Constraints** [Witkin and Kass,1988], and serve as the foundation for many modern CMS research. Jain et al. [2009] provides an example for locomotion; Macchietto et al. [2009] find a method for balance maintaining movement. Liu [2009] proposed a method for object manipulating animation.

The Spacetime method may modify the motion trajectory and in nature it solves the problem through vibrational optimization. However it faces several key problems. **Efficiency** In many cases, it will take very long time to find the “best” solution and there is no guarantee the optimal solution can be achieved. And for complex body structures the computation will takes prohibitive long time [Anderson and Pandy, 2001]. Optimization techniques like time window and multi-grid techniques are proposed by Cohen [1992] and Liu et al. [1994]. Because of the computational burden, very a few researchers [Popovi´c and Witkin, 1999] proposed Spacetime Constraint for full body dynamic animation. **Sensitive and Overspecific** Current numeric methods are very sensitive to model accuracy and initial conditions. Precise model for both the environment and body have to be prebuilt. Liu [2005] points out that spacetime constraint methods only suit high energy motions like jumping and running; for low energy motion tasks like walking the result doesn’t looks natural. This is mainly because the muscle effects are neglected. Motions like heart beating, breathing, or motions of other animals such as the swimming of fish and jellyfish, flying of birds have not been synthesized with dynamic methods for the lack of a feasible dynamic model.

2.2 Biological Research

In biological research, motor control is an age old problem full of paradoxes. Motor control in nature is a complex process involving many chemical, electrical and mechanical effects. As a result, most of the dynamic methods involve complicated computation. However this is very opposite to the characteristics of the neural systems of real creatures. **Time Delay** Neural signal transmitting speed is very slow; and there is a long delay between neural signal firing and force generation in muscles. **Noisy** Besides the delay and low speed transition, the neural signals are also noisy. The body structure and environment are also nonlinear, noisy and time varying. **Limited Activity** Current research evidences and common life experience show that motor control involves little control effort. Many experiments show motion can happen even without brain input.

Despite the complexity of body structures and environment, the natural motor control strategy seems relatively simple involving little computational work. In many animals, the active neural structure in motor control is the **Central Pattern Generator (CPG)** which generates rhythmic signals. There are many experimental researches in robotics and biomechanics succeeded in controlling some motion with very simple strategy [Nishikawa et al., 2007]. **Uncontrolled Manifold Hypothesis** method even proposed that some DOFs are not controlled and freely influenced by the environment [Latash, 2008]. **The Equilibrium Point Hypothesis** suggests that what the neural systems controls is not trajectory, but the final position. **The Impedance Control Hypothesis** [Hogan, 1985] method refines the idea of EPH by providing an explanation for effects of the extra DOFs. Impedance Control proposed the extra DOFs provide a way to control the stability and admittance of final position according to the motion purpose. Morphological Computation Theory [Nishikawa et al., 2007, Pfeifer and Iida, 2005] thinks both the body structure and the environment play a crucial role in motor control, basic motion patterns are generated by body and environment, the neural systems only maintains or tweaks such motion patterns.

The biological ideas provide space for an efficient motion adaptation, but the theory are incomplete and mainly for explaining experiment results. There is a big knowledge gap to turn it into a sound control theory.

3. QUALITATIVE CONTROL THEORY FOR CMS

Inspired by the biological research, in this paper, a different strategy for motion adaptation is proposed. The key idea is that the environment is allowed to affect the motion freely; control effort is only applied when the qualitative properties of motion are violated.

3.1 The Qualitative Control Theory

The Qualitative Control Theory is a mathematical description of the Morphological Computation Theory. In qualitative control theory the basic patterns of motion are called **motion primitive**. In mathematic terms, motion primitives are **structural stable**.

3.1.1 Basic Concepts of Qualitative Dynamics.

The configuration of system is described using state value in the state space. We use vector q to represent the state of a system, M is the state space which is a manifold, the motion trajectory over time is $q(t)$. For a dynamic system, $q(t)$ is usually represented in the form of ordinary differential equation.

$$\dot{q} = F_u(q), q \in M \quad (1)$$

where u is the control effort. F is determined by the system's natural property. If $u = 0$, no control effort is applied. Such systems are autonomous systems. For every point $q \in M$, F and u determines a derivative vector $\dot{q}$. All the vectors over the full space of M form the vector field. There is a corresponding geometry structure for Equation (1), a differentiable manifold. The motion trajectory can be found by apply the integral operation on the vector field. The result trajectory is defined as flow Φ, all the flows form another geometrical structure, the phase portrait, which illustrates all the possible motions of the dynamic system.

On the phase plane, flows can only intersect at some special position.

Fix Point, where $F(q) = 0$. **Period Flow** For any point q on the circle, we have $F(q(0)) = F(q(T))$.

Intersections like fixed point are also called **equlibria**. At each equilbria, the local space can be divided into three subspace of submanifold: centre submanifold, stable manifold, and unstable submanifold. For nonlinear system, globally, the shape of stable and unstable submanifold may be bending and connect with itself or each other. The equilibra and its connectivity of sub manifolds form a topological structure. The phase plane is divided into different regions, result in a cellular structure. In each region, there is only one attractor, all the flow in this region will converge to the attractor, and the corresponding region is called **basin of attraction**.

3.1.2 Motion Adaptation under Qualitative Control

A Mechanical system can be extremely stable without any control effort. This kind of stability is rough stability or structure stability [Andronov and Pontryagin, 1937] which is determined by the topology structure of the system [Jonckheere, 1997]. Using Qualitative Control, motion will be defined by the topological structure of the corresponding differential equation. Motion adaptation can be modeled as homeomorphism. Homeomorphic flows can be generated if the differentiable manifolds are homeomorphic, which means they share the same topological structure, but with different shapes. Structure stable autonomous systems have the ability to maintain its topology structure under perturbations, thus the resulting motion is adaptive but qualitatively unchanged.

3.1.3 The Ship Example

Here we take the simple floating ship example to show the idea of structural stability. In real life, typical ships have bigger height than width, as shown in Figure 1.

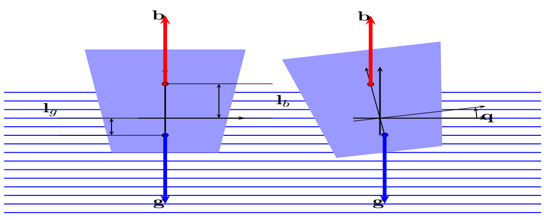

Figure 1. The Floating Ship Example

How the ship maintains its configuration or “posture” is a question. Through analyzing the topology and structural stability, we show that it requires little effort to maintain this posture. This conclusion applies to different ships since their dynamics are qualitatively the same, or topologically conjugate. Its motion is determined by the torques of gravity g, buoyancy b and external control. A ship will only rest at the postures where the sums of torques are zero, the Equilibrium points. The only two possible ones are shown in Figure 2.

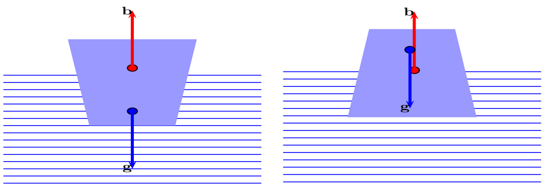

Figure 2. The Stable and Unstable Posture

The left posture in Figure 2 is attractive or stable, whereas right posture in Figure 2 is repelling or unstable. All the flow curves start from the unstable position and terminate at the unstable position. It has the topology structure as shown in Figure 3. This means that no matter what the current posture, the ship will return to the normal stable posture automatically.

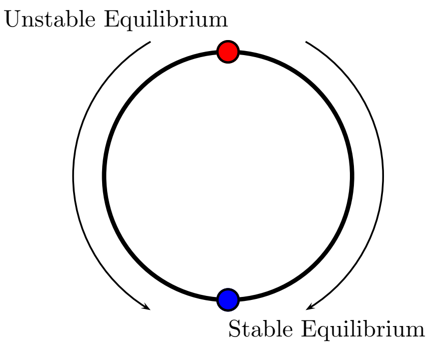

Figure 3. The Topology Structure of the Ship

This conclusion is independent of the shape, size, weight or material of the ship. In general cases, the same wave perturbation will result in different sway motions for different ships. However, as long as the qualitative structure design criterion is maintained, balancing remains “easy”. In mathematical terms, all the phase portraits share the same topological structure of Figure 3.

3.2 Motion Synthesis based on Qualitative Control

In our method, only the final motion is concerned. In mathematical viewport, only the attractors of flows are controlled, while the flow shape is not considered in motion control. So according to the types of attractors, motion can be categorized into two groups. **Discrete Motion** Such motions have fixed attractors. Typical motions include posture control and picking up motion of the arm. **Periodic Motion** Such motions have periodic attractors, typical motion include walking, running and heart beating.

Motions are made up of motion primitives. Neural control system only tweaks the basic motion primitives to achieve specific objective. According to qualitative control theory, our approach will preserve the three natural motion features for the following reasons. **Adaptive** Different perturbations will result in different shapes of the manifold, motion will be changed according to the environment. **Efficient** Motion will be generated passively and follow the least energy path. **Agile** High precise calculation not needed, topological structure can be manipulated and maintained by some very simple computation.

3.3 The New Control Scheme from Qualitative Control Theory

An animal's body and environment can be extremely complex. This usually leads to high dimensional manifolds with complicated topological structure. Many CMS research have asked the same question whether such complex system can be controlled with a simple method. Biology Research suggested that the motion is mainly controlled by the Central Pattern Generator (CPG), the autonomous network that generating rhythmic signals. The existence of CPG is very common, from primitive animals like lamprey and fish, to high level animals like bird, mammal and human [Cohen, 1988]. We think that motor control by rhythmic signals can be modeled as entrainment [Gonzalez- ´Miranda, 2004]. Based on Qualitative Control Theory, in this section, we will discuss a new control scheme using biological entrainment.

3.3.1 The Biological Entrainment

Entrainment is the phenomenon that two coupled oscillator systems oscillate in a synchronize way. Although the mechanism can be very complex, the phenomenon is universal. Entrainment will happen when coupling two oscillators with similar oscillation frequencies but with very different characteristics. A simple explanation is that energy fluctuates between the two oscillating system. For some cases, stability can be enhanced and chaotic behavior can be suppressed.

Our new control scheme is based on the entrainment. The neural system form one electrical oscillator; body and environment form the other mechanical oscillator. Mechanical oscillator can be controlled by the oscillation property of the neural system through entrainment effects. The property of neural oscillator will greatly affect the mechanical motion results.

3.3.2 The Structural Stability of Neural Oscillator

One extensively studied oscillation model is developed by Matsuoka [1985]. The mathematical presentation is as follows:

$$
\begin{gathered}
\tau_1 \dot{x}_1 = c - x_1 - \beta v_1 - \gamma [x_2]^+ - \sum_j h_j [g_j]^+ \\
\tau_2 \dot{v}_1 = [x_1]^+ - v_1 \\
\tau_1 \dot{x}_2 = c - x_2 - \beta v_2 - \gamma [x_1]^+ - \sum_j h_j [g_j]^- \\
\tau_2 \dot{v}_2 = [x_2]^+ - v_2 \\
y_i = \max(x_i, 0) \\
y_{out} = [x_1]^+ - [x_2]^+ = y_1 - y_2
\end{gathered}
\tag{2}
$$

where x and v are state variables of the oscillator, τ,c,β,γ are parameters of the oscillator.

Early Research shows that Matuoka oscillator is autonomous oscillator and adaptive; Entrainment behavior can happen when couple it with different oscillators. But because of the nonlinear properties, its behavior is not completely understood. Matsuta[1987] explains the adaptive properties from the location of the roots of characteristic equation. Wilimas[1998] explains the properties in frequency domain.

Here we provide an idea about structural stability from the topological viewport. Basically, neural oscillator shows three important properties: **Simple Structure** The topology structure of neural oscillator is simple; it includes one attractive limit circle and one fix repellor. **Large Basin of Attraction** All the simulations we carried out converged to the same limited circle. **Fast Converging Speed** In most of the case, the flow will converge to the limit circle within one period time. Features above are shown in Figure 4.

Through this example, we believe neural oscillator is structure stable. The large area of basin of attraction means the final behavior is totally determined by parameters. Initial conditions will have no effects on the final oscillation. The converging speed can be seen as quick recovery ability. When an impulse perturbation happens, it will recover in one period time. These properties are very valuable in CMS research. An intuitive idea is that we couple the neural oscillator with mechanical oscillator of body and environment, thus make the motion structural stable.

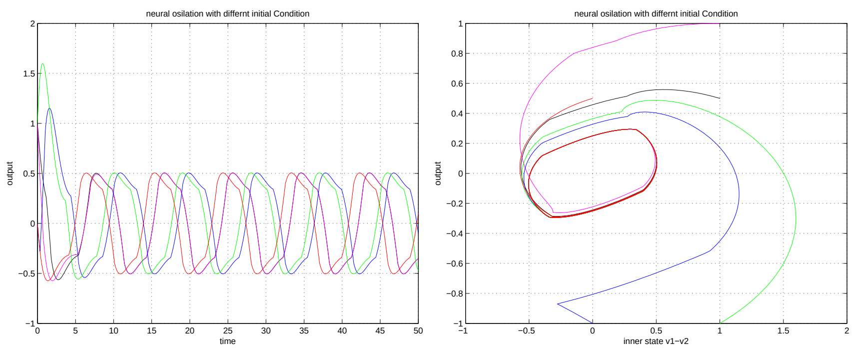

Figure 4. The state plot and phase plot of Matsuta Oscillator

4. APPLICATION AND RESULTS

The basic idea of using qualitative control for motion synthesis is to let body and environment form a basic oscillation pattern and use the neural oscillation to boost structural stability. Our approach can be applied to many motion tasks.

In this section, we will discuss just one example in details, the bipedal walking. This is mainly because bipedal walking is one of the most challenging and common locomotion styles. From the mechanical perspective, bipedal walking is unstable which makes it very difficult for adaptive gaits. It needs special care in the control system design.

Based on the biology research, walking involves little reasoning activity. The number of neurons that take part in the lower limb control is very limited, much less than arm, hand and even tongue. While for artificial system, robust bipedal walking is difficult to achieve. Many control method has been tried, but none of them shows comparable performance with human walking.

In dynamic research, natural looking gaits can be generated by passive method. There have been a series of passive dynamic walking machine [McGeer, 1990a,b]. If we put a passive walking machine on a slope, without any effort, it can walk down the slope. However the stabilities are very fragile. Passive walking can only be maintained when walking down a specific slope under specific condition.

From the Qualitative Control Theory, we can see the real reason why passive walking machines can walk down the slope. It is because the existence of a limit circles for the dynamic interaction between body and ground. The fragile stability means the basin of attraction covers only a small area on the phase plane. For natural looking walking motion, we plan to boost the stability of the passive walking machine by neural oscillation entrainment.

4.1 2D Passive Walking Model

The mechanical model we adopted is illustrated in Figure 5. Passive walking is not a continuous dynamic system. We separate the motion into two phases and formulate two equations. **Leg Swing Phase** During the swing phases, we suppose that one leg is fixed on the ground, the arc foot makes the passive dynamic walker rolling without sliding. The equation is in the form of Equation 3. Where $q = [x, y, \phi_1, \phi_2]$ is the generalized coordinates, $\overline{M}$ is the Mass Matrix, D is the constraint matrix, F is the external force, F_c is the constrained force. For details of each component, please refer to the work [Wisse and Schwab, 2005]. **Heel Strike Phase** We suppose the heel strike the ground in a short time, the angular momentum is preserved.

$$\begin{bmatrix} \overline{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \bar{F} \\ \ddot{D} \end{bmatrix} \tag{3}$$

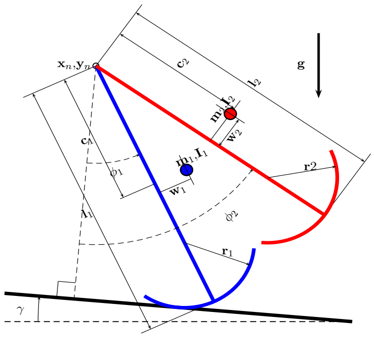

Figure 5. Passive Walking Model

4.2 Adaptive Walking Motion

The input of neural oscillator G_{input} is defined by the difference angle between the two legs.

$$G_{input} = \phi_1 - \phi_2$$

Neural output will drive the biped walker. After adding the neural control, the equation of the dynamic system is

$$\begin{bmatrix} \bar{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \bar{F} \\ \ddot{D} \end{bmatrix} + \begin{bmatrix} U \\ 0 \end{bmatrix} \quad (4)$$

Where U is the control signal generated by the neural oscillator. Neural oscillator output G_{out} is applied at the hip joint to actuate the two legs towards different directions.

$$U = [0,0,1,-1] * G_{out}$$

Passive Walking When the passive walker walks down a slope, for every step, there is energy input from the potential energy, and there is also energy loss because of heel strike. There must be an equilibrium condition when the energy lost is equal to the energy input. Because there is no extra control energy input, such motion is the most energy efficient. Figure 6 shows the gait of the passive walker. After coupling the neural oscillator, the basic pattern is not changed as shown in Figure 7.

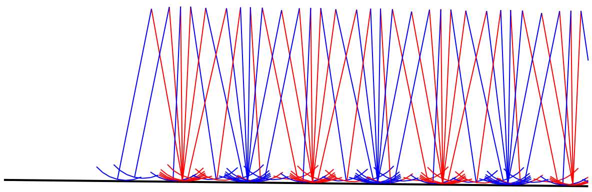

Figure 6. Passive Gait

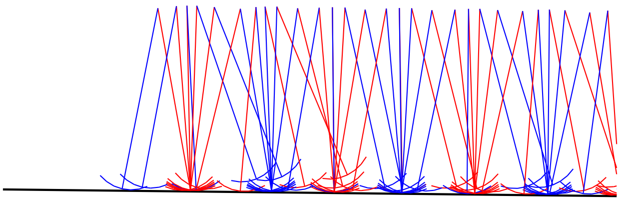

Figure 7. Gait with CPG entrainment

Walking On Plain However the stability of this passive waking is fragile. The passive walker can't walk on plane. The step size will decrease after each step, and finally it will stop or fall over as illustrated in Figure 8. After coupled with the neural oscillator, this walking machine can walk on plane, and exhibits gait similar to the passive dynamic walker. Figure 9 shows the gait. From the state plot and phase plot in Figure 10, we can see that the gait converged to a stable limit circle.

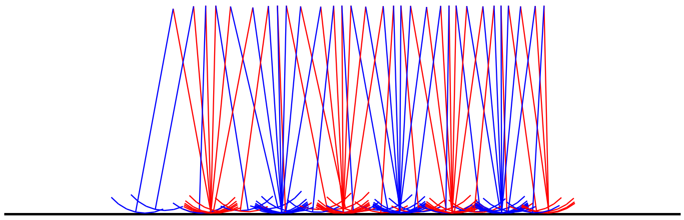

Figure 8. Passive Gait on plain

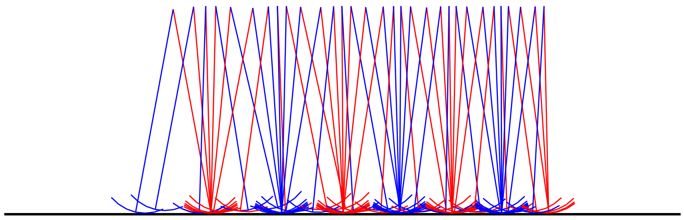

Figure 9. Gait of Entrainment on plain

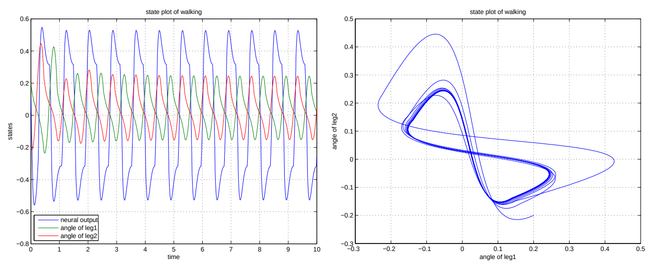

Figure 10. the state plot and phase plot of walking on plain

4.3 Structural Stability under Perturbations

To verify the structural stability, we introduce a variety of perturbations to the passive walker. These perturbations include different initial condition, different slopes, different leg mass and different leg length.

Different Initial Condition The original passive walker is not very stable. A slight change in initial condition will result in walking failure. While after coupled with neural oscillator, the basin of attraction has been enlarged. A different initial condition can still lead to a stable gait, as show in Figure 9. Natural looking gait is maintained.

Walking On Different Slopes Another parameter we change is angle of the walking slope. When we increase the down slope, stable walking motion can still be maintained, as shown in figure 11. An important discovery is that although the walkers can walk on various down slopes, it cannot walk up slope, no matter how control parameters are changed. It can't walk up slope and will fall backward after several steps. We think that this is mainly because the proper limit circle does not exist in the dynamic system when walking up slope. Involving the upper body into this structure may help to solve this problem.

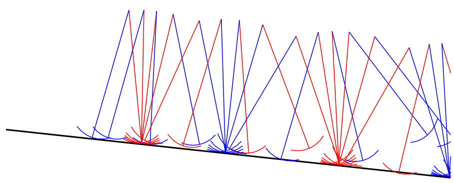

Figure 11. Gait on a Big Slope

Leg Mass Variation We add mass on one leg to 150% and find the stability of the gait is still maintained. The step length and swing period of the two legs are different, this gait is similar to that with a crippled leg, see Figure 12.

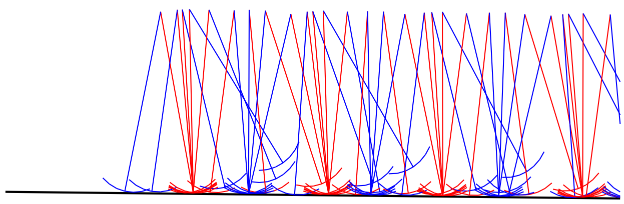

Figure 12. Crippled Gait because of Different Mass

Leg Length Variation The last parameter we change is the leg length. We change the leg length to 1/8 shorter. And we find the stability of the gait is maintained, see Figure 13.

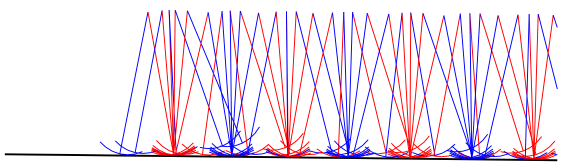

Figure 13. Gait with Shorter Legs

5. CONCLUSION AND FURTHER WORK

Qualitative Control Theory can synthesize motion with adaptive behavior while keeping the qualitative properties of motion. It provides a new method to synthesize adaptive motion efficiently. Since very little computation involved in each controller, compared with traditional optimization based method, this method can generate motions in real-time. And most importantly, our method is parallel in nature. Each CPG only control on single degree of freedom. For complicate characters, many different CPGs can be simulated in parallel without referencing each other. Since many physical simulation modules have been implemented efficiently using GPU, in future, most of the computational burden of our method can be shifted to GPU. This will make our algorithm generating agile motions even with very complicated environment and involving whole body structures.

However since we bring in a new theory into the motion synthesis area, many works need to be done in the future. For example, our current model only involves the lower body structure, upper body and more joints will be considered in our future design. To proof the adaptions, we will need experiment on more complicate terrain instead of just upslope and downslope.

More Central Pattern Generators are needed for different kinds of motions. And how to turn the CPG parameters for the animator purpose are still open. These topics will be covered in the future research.

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