SKETCH-BASED MODELING SYSTEM WITH CONVOLUTION AND VARIATIONAL IMPLICIT SURFACES

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ABSTRACT

In this work we focus on function representation suitable for creating 3D freeform shapes with sketched silhouette curves. We aim at comparison of two approaches for sketch-based modeling, namely the skeleton-based convolution surfaces and variational implicit surfaces. Both methods are extended to handle smooth set theoretic operations between components defined by sketching silhouette curves on different projection planes. Our approach includes additional extensions to sketch-based systems like automatic skeleton generation that can be directly used for object animation, carving operation, creating surfaces with handles and surface texturing.

1 INTRODUCTION

Sketching is a very natural process for most people, it makes the design faster and intuitive. In this work we focus on designing a 3D freeform shape from sketched silhouette curves supporting natural human abilities and offering the rich expression afforded by pen movements.

First freeform sketched-based modeling system Teddy [4] used polygonal mesh as geometric representation. In recent years it has been, in most of the freeform user interfaces, replaced by implicit surface representation because polygonal representation has limitations on surface smoothness and fairness. Implicit functions allow to introduce new interesting operations on freeform models like blending union or carving that are very difficult to implement with polygonal meshes.

Owada et al. [6] proposed a similar interface as the Teddy system but instead of polygonal representation they used volumetric representation. Volumetric representation can achieve smooth result but at the cost of higher storage and also higher computation time. Turk et al. [11] first adopted the variational implicit surfaces for object modeling. The main problem of their approach is the difficulty of reconstruction of the 3D shape thickness. The method works well for ellipse-like shapes other shapes hard to create. One of recent freeform user interfaces that use skeletons is system ConvMo [10] which works based on cylindrical convolution surfaces. A weakness of their approach is that it produces unwanted bulges. Above disadvantages of ConvMo system were partially eliminated

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in [1] by representing the surface as blended spheres, placed along the skeleton similar to [12]. Spheres have smaller regions of influence as convolution cylinders, so they follow more faithfully the shape of silhouette curve.

Inspired by the above approaches we have developed a system consisting of two approaches the skeleton based convolution surfaces and variational surfaces to reconstruct a 3D shape from user defined silhouette. By comparing of two approaches we conclude that the mixture of both methods for different parts of the shape should work best of all. In our first approach we reduce unwanted bulging effect by simplifying the skeleton and diminishing of field contribution at skeletal joints. Since this approach is not enough to remove the effect, we proposed the global interpolation with initial solution found by local interpolation step. Moreover, we propose the method for reduction of constraints thus reducing the computation costs of variational implicit surface and unwanted oscillations on surfaces. At the end we propose several operation for shape manipulation that are robust and very simple to implement.

The paper is organized as follows. In Section 2 we discuss the previous work on function shape representation. The proposed methods are discussed in Section 3 We demonstrate the results obtained using our techniques in Section 4and we conclude this paper in Section 5

2. SHAPE REPRESENTATION: F-REP

In the process of sketching the objects the algorithms must perform a variety of geometrical calculations, therefore the general mathematical representation is crucial shape representation. We have focus on function representation based on two defining functions, namely convolution surfaces generated by skeletons and variational implicit functions. The convolution surfaces generated by skeletons are well suited for animatable shapes, since they can be stored it in a very compact way. On the other hand, variational functions have the benefit of locally defined field functions to optimize the modeling process; unfortunately, such shapes have no skeleton and thus are not suited for animation.

Let us consider closed subsets of n-dimensional Euclidian space E^n with the definition:

$$f(x_1, x_2, ..., x_n) \geq 0,$$

where f is a real continuous function defined on E^n. The above inequality is called a function representation (*F-rep*) of a geometric object and function f is called the *defining function* [7].

2.1 Shape from convolution cylinders

Let us consider from now on the defining function given by the convolution operator between a kernel and all the points of a skeleton. A skeleton-based implicit surface generated by a set of skeleton primitives $r_i, i = 1, ..., n$ with associated energy function F_i is commonly defined as an iso-surface satisfying the following equation:

$$S = \left\{ \mathbf{p} \in R^3 : \sum_{i=1}^{n} F_i(\mathbf{p}) = T \right\}.$$

Parameter T is the iso-potential threshold value and energy functions F_i are monotonically decreasing functions of distance $\mathbf{r}_i$ from the skeleton primitive: The shape of a convolution surface can be varied in several ways: by changing the skeleton, the modifying the convolution kernel, and by spatial deformation of convolution surface. Jin at al. proposed a polynomial weighted kernel along line segment, arc, and quadric curve

$$F(\mathbf{p}, \mathbf{c}(\cdot), w(\cdot)) = \int_0^l \frac{w(t)}{(1 + s^2 r^2(t))^2} dt,$$

where $\mathbf{c}(t)$, $0 \leq t \leq l$ is a curve, $w(t)$ is a weight and $r^2(t) = \|\mathbf{r} - \mathbf{c}(t)\|$. Authors also derived the analytical solution of the integral.

2.2 Shape from variational functions

Variational implicit surface is an interpolative function, f, satisfying:

$$f(\mathbf{c}_i) = \begin{cases} 0 & \text{for each known surface point } \mathbf{c}_i, \\ 1 & \text{for } \mathbf{c}_i \text{ inside the surface,} \\ -1 & \text{for } \mathbf{c}_i \text{ outside the surface.} \end{cases}$$

Thinking about points $\mathbf{c}_i$ as constraints we can formulate the following scattered-data interpolation problem:

Problem 1 *Given a set of k constraint points $\mathbf{c}_1$,..., $\mathbf{c}_k$ scattered over xy-plane, with associated scalar height values h_1, ..., h_k, construct a surface that interpolates the constraints $(\mathbf{c}_i, h_i)$, e.g. $f(\mathbf{c}_i) = h_i$ for $i = 1, \ldots, k$.*

The thin-plate solution to this problem is the function $f(\mathbf{x})$ that satisfies the constraints and minimizes the energy function $E(f) = \int_\Omega f_{xx}^2(\mathbf{x}) + 2f_{xy}^2(\mathbf{x}) + f_{yy}^2(x)d\mathbf{x}$, where f_{xx}, f_{xy}, and f_{yy} are second partial derivatives. We can thing of f as a linear combination of radial basis functions (RBF) centered at constraint points $\mathbf{c}_i$. General RBF can be written in form $f_i(\mathbf{x}) = \phi(\|\mathbf{x} - \mathbf{c}_i\|)$ that depends only on the distance between argument point $\mathbf{x}$ and RBF center $\mathbf{c}_i$. Taking into account the linear and constant part of the solution we can write the resulting interpolated function as

$$f(\mathbf{x}) = \sum_{i=1}^{k} w_i \phi(\|\mathbf{x} - \mathbf{c}_i\|) + P(\mathbf{x}),$$

where $P(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + b$, $\mathbf{a} \in \mathrm{R}^3$ and $b \in \mathrm{R}$. The problem is to determine weights w_i and coefficients of $P(x)$ which yields to the system on linear equations:

$$\begin{bmatrix} \phi_{11} & \phi_{12} & . & \phi_{12} & 1 & c_1^x & c_1^y & c_1^z \\ \phi_{21} & \phi_{22} & . & \phi_{22} & 1 & c_2^x & c_2^y & c_2^z \\ . & . & . & . & . & . & . & . \\ \phi_{k1} & \phi_{k2} & . & \phi_{k2} & 1 & c_k^x & c_k^y & c_k^z \\ 1 & 1 & . & 1 & 0 & 0 & 0 & 0 \\ c_1^x & c_2^x & . & c_k^x & 0 & 0 & 0 & 0 \\ c_1^y & c_2^y & . & c_k^y & 0 & 0 & 0 & 0 \\ c_1^z & c_2^z & . & c_k^z & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ . \\ w_k \\ b \\ a^x \\ a^y \\ a^z \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ . \\ h_k \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\mathbf{c}_i = (c_i^x, c_i^y, c_i^z)$ and $\phi_{ij} = \phi(\|\mathbf{c}_i - \mathbf{c}_j\|)$. Last four rows in the system correspond to orthogonality conditions:

$$\sum_{j=1}^{k} w_j = \sum_{j=1}^{k} w_j \mathbf{c}_j^x = \sum_{j=1}^{k} w_j \mathbf{c}_j^y = \sum_{j=1}^{k} w_j \mathbf{c}_j^z = 0.$$

The associated linear system will always have a solution, therefore it can be solved by iterative methods or SVD decomposition.

The last thing that should be done is the definition of constraints. To simplify the process Turk [11] proposed four different constraints:

1. *Boundary constraints* $(\mathbf{c}_i, h_i)$ verify that $f(\mathbf{c}_i) = h_i$ with $h_i = 0$ and the surface must pass through points $\mathbf{c}_i$.
2. *Interior constraints* $(\mathbf{c}_i, h_i)$ verify that $f(\mathbf{c}_i) = h_i$ with $h_i > 0$. Points $\mathbf{c}_i$ are located arbitrary inside the surface.
3. *Exterior constraints* $(\mathbf{c}_i, h_i)$ verify that $f(\mathbf{c}_i) = h_i$ with $h_i < 0$. Points $\mathbf{c}_i$ are located arbitrary outside the surface.
4. *Normal constraints* $(\mathbf{c}_i, h_i)$ specify the surface normal and are located outside the surface at a small distance from a boundary in direction $\vec{\mathbf{n}}$. They verify that $f(\mathbf{c}_i + \varepsilon\vec{\mathbf{n}}) = h_i$ with $h_i < 0$.

3. RECONSTRUCTION OF SURFACE FROM SILHOUETTE CURVE

In this section we describe the main steps for creating convolution and variational implicit surfaces from sketched silhouette curves. Both methods have some common steps but they differ in technique used for extracting the data needed for shape reconstruction from the silhouette curve.

3.1 Skeleton based surface

The following method summarizes the main steps of skeleton based convolution surface reconstruction from sketch:

1. *Process the input stroke and extract the skeleton from its simplification.*
2. *Convolution surface construction by local fitting of weighted kernel along the skeleton segments to the silhouette curve.*
3. *Global fitting of convolution surface to the silhouette by least-square method.*
4. *Modify the generated surface components by merging, carving, and other editing operations, refer to Fig. 1.*

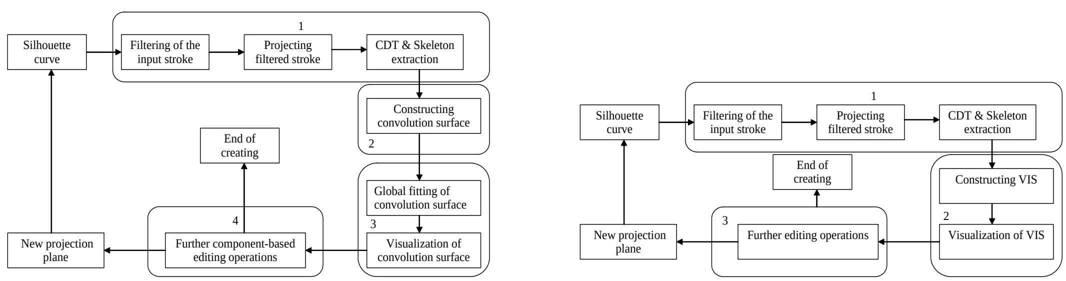

Figure 1. Basic steps. Left: Skeleton based convolution surface generation from sketch. Right: Implicit surface construction from sketch.

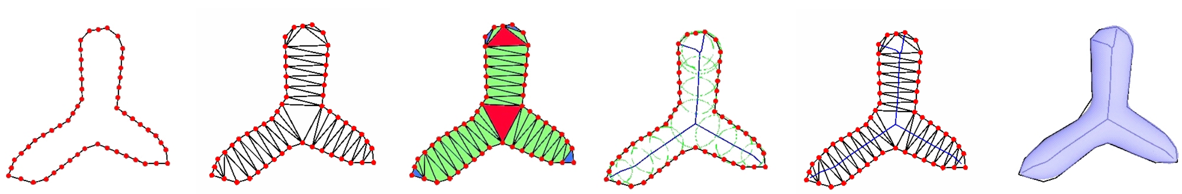

Figure 2. Process of generating convolution surface from silhouette. From left: Filtered silhouette curve; result of CDT; terminal (blue), sleeve (green), and junction (red) triangles, influence circles; simplified skeleton; reconstructed convolution surface.

3.1.1 Processing the input stroke

Filtering the input stroke. At this step the silhouette curve drawn by the user is converted into a simple polygon by sampling the input device movement. Several different algorithms can be found for point reduction in a polygon.

In general they can be calcified into two groups those dependent on screen resolution using the edges with uniform length [4] and those using adaptive edge length [3, 10]. We use a modified Douglas and Peucker algorithm [3]. Original nonuniform distribution of points along the silhouette is re-sampled to space the points uniformly on simplified polygon, see Fig. 2. All simplified strokes with self-intersections are rejected at this stage and user is requested to draw a new stroke.

Projecting filtered stroke. At this point we transform the stroke from screen coordinates to 3D coordinates by projecting it to a projection plane. Projection plane is parallel to screen plane passing through 3D position of a first silhouette point obtained from camera depth, see Fig. 3.

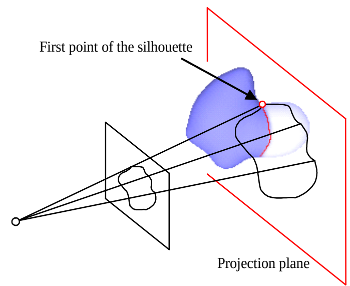

Figure 3. Projection of filtered stroke.

Skeleton extraction. At this step a good skeleton must be found to get the best guess of surface close to the silhouette. Similar to chordal axis transform [8], first, the constrained Delaunay triangulation (CDT) is applied, where constrains are the edges of filtered polygon. Second, triangles are claciffied into three categories, see Fig. 2 top raw:

- Terminal triangle (T-triangle) is a triangle with two external edges (marked in blue),
- Sleeve triangle (S-triangle) is a triangle with one external edge (marked in green),
- Junction triangle (J-triangle) is a triangle that has no external edges (marked in red).

To find a good initial guess an algorithm based on medial axis comprising of line segments should be used as a skeleton. Unfortunately this approach will produce unstable results for so called ”folding” strokes [10].

We propose the skeleton extraction algorithm which unlike [10] uses not only circumscribed circles of triangles but rather for T-triangle it uses inscribed circles with their incenters; for J-triangles it uses circumscribed with circumcenters; and for S-triangles it uses either circumscribed circles or circles with external edge of polygon as tangent and third vertex of triangle as point of circle.

3.1.2 Constructing a convolution surface.

To find the convolution that fits given silhouette, we need to determine parameters of its field function $F(\mathbf{r})$. The first approximation to $F(\mathbf{r})$ is computed by considering each skeleton segment being isolated, neglecting the influence on other elements. In such a case for a single line segment with end points $\mathbf{p}$, $\mathbf{q}$ and length l we assign weights w_0 and w_1 at the end points by setting the field contribution at $\mathbf{p}$ to $\mathbf{r_p}$ at distance $R_\mathbf{p}$ and at $\mathbf{q}$ to $\mathbf{r_q}$ at distance $R_\mathbf{q}$, see Fig. **??**. $R_\mathbf{x}$ denotes the distance from center $\mathbf{x}$ to the point where field function is equal to the threshold T, e.g. $F(\mathbf{r_p}) = F(\mathbf{r_q}) = T$. Last equation can be written as

$$\begin{aligned} lT &= w_0(lF_l(\mathbf{r_p}) - F_t(\mathbf{r_p})) + w_1 F_t(\mathbf{r_p}), \\ lT &= w_0(lF_l(\mathbf{r_q}) - F_t(\mathbf{r_q})) + w_1 F_t(\mathbf{r_q}). \end{aligned}$$

Because $F_t(\mathbf{r_p})$ and $lF_l(\mathbf{r_q}) - F_t(\mathbf{r_q})$ are small we get the estimation of weights as follows:

$$w_0 = \frac{lT}{(lF_l(\mathbf{r_p}) - F_t(\mathbf{r_p}))},$$
$$w_1 = \frac{lT}{F_t(\mathbf{r_q})}.$$

Unfortunately, the resulting surface has a bulge in the middle. We use similar idea as in [10] to ensure that the surface passes through the point $\mathbf{r_{mid}}$, which is at the distance $(R\mathbf{p} + R\mathbf{q})/2$ we scale the weight w_0 and w_1 by a correction factor of $\kappa = T/F(\mathbf{r_{mid}})$.

Finally, the field function of resulting convolution surface consisting of several linear skeletal elements can be obtained by summing the field function for all line segments L_i, $i = 1, 2, \ldots, n$: $F(\mathbf{r}) = \sum_{i=1}^{n} \lambda_i F(\mathbf{r}, L_i)$, where $lambda_i$ is control scale factor. In the first approximation step $lambda_i = 1$.

3.1.3 Global fitting of the convolution surface.

With $lambda_i = 1$ and weights w_0, w_1 computed as described above, one could notice that the surface does not interpolate the silhouette curve, see Fig. 4 top. This is caused by the fact that when field contribution of all segments are summed the resulting surface begins to inflate. This inflating effect is fully suppressed in global interpolation step. We solve the constrained least-squares problem for unknowns $lambda_i = 1$ while minimizing the distance between the convolution surface and the silhouette polygon points.

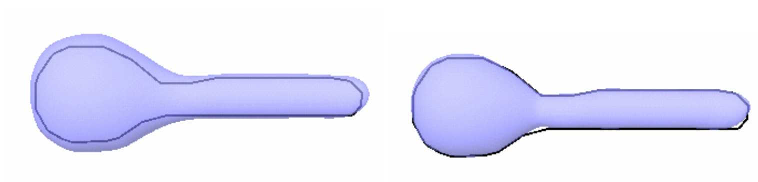

Figure 4. Surface without and with global interpolation.

3.1.4 Operations

Merging of two components is for free because two field functions are summed up to form a smooth surface. In order to have larger control over the blending of two components the F-rep blending union operation is implemented.

Carving operation is also implemented as F-rep blending substraction operation.

Surfaces with handles can be created by substraction operation between two components. Hoverer, there is a way create such surfaces directly form two silhouettes one inside the other one. Our algorithm can handle such cases and the result will be a torus.

3.2 Variational implicit surface

The main steps of a method for reconstruction of variational implicit surfaces (VIS) are as follows:

1. *Process the input stroke and extract the skeleton from its simplification.*

2. *Construction if variational implicit surface by definition of boundary and normal constraints. Final VIS then matches the silhouette curve.*

3. *Global fitting of convolution surface to the silhouette by least-square method.*

4. *Modify the generated surface components by merging, carving, and other editing operations, refer to Fig. 1.*

3.2.1 Processing the input stroke

The first step of VIS reconstruction method from sketched silhouette curve, processing of the input stroke, is similar to the steps from Section 3.1.1. First two step *Filtering the input stroke* and *Projecting filtered stroke* are exactly the same steps as in Section 3.1.1.

Skeleton extraction. At this step a good skeleton must be found with the height value for each skeleton vertex. The third step is a bit different. To find the skeleton we use our proposed method from above, this guaranties us non-redundant constraints for skeleton vertices. This step is different from commonly used approaches based on VIS where they had many constraints and big matrices [2]. Finally, we assign the height value to each skeleton vertex equal to the average distance from the sketched polygon vertices in the neighborhood. By this approach we have eliminated unwanted surface oscillations.

3.2.2 Constructing a variational surface

Construction of variational implicit surface requires the specification of constraints. For each silhouette point we compute the outside normal to the silhouette polygon. Then a normal constraint is defined by shifting the boundary constraint placed at the silhouette point by ε in normal direction. Such normal constraints all lie in the silhouette plane. For each skeleton vertex we generate two normal constraints, one above and one below the silhouette plane.

After defining these constraints we have all necessary information to create the linear system from Sec. 2.2. The resulting function interpolates the constraints and no additional step are needed to correct the solution.

3.2.3 Operations

Carving operation wasn't proposed in previous sketch-based systems based on VIS, even though the mathematical concept is very easy. Similar, to merging algorithm, we eliminate the constraints of the carved component which lie in the intersection of two input components and of eliminating the constraints of carving component which lie outside the carved component. The constraints of carving object that remained are reversed, see Fig. 5.

In order to have better control over the blending of two components the F-rep blending substraction operation can also be implemented.

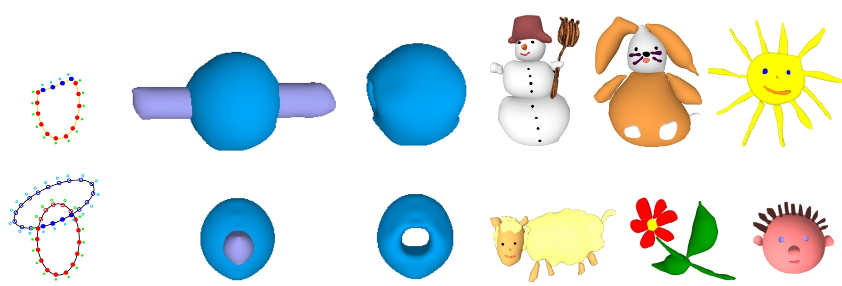

Figure 5. VIS carving. Left: The constraints of carving object that remained in use are reversed. Center: Carving example. Right: Examples of various objects created in SketchCo.

4 COMPARISON OF PROPOSED METHODS

In this chapter we want to compare two methods for creating 3D freeform shapes from 2D sketched silhouette curves proposed above. Namely, possibilities and limits of skeleton based convolution surface and variational implicit surface for sketch-based modeling.

In the case of VIS, the resulting 3D freeform surface can be generated directly from silhouette points because VIS are specified by a set of boundary or normal constraints. Therefore it is sufficient to sample input stroke and sample its points for constrain definition. On the other hand skeleton based convolution surface needs to find the skeletal elements for their definition, however such objects are directly suitable for animation.

Moreover, robustness of resulting linear system strongly depends on the set of constraints, the change of a single constraint can change the result significantly. Considering the shape manipulation, VIS allows very easy to implement basic operations, however, when more control is needed over the operation (such as blending of shape) skeleton based representation suites better the purpose.

5 CONCLUSION

Proposed methods were implemented as SketchCo modeling system. Models in Figure 5 were created in 6 minutes by children from 6 to 15 years. We have considered several geometric representations for sketch-based modeling. Finally, we have proposed several modifications of two implicit surface representations, the skeleton based convolution surfaces with linearly weighted distribution and variational implicit surfaces with few editing operations. In our first approach with skeleton based convolution surfaces we reduced unwanted bulging effect that appears around junctions of blend line segments by simplifying the skeleton and diminishing of field contribution at skeletal joints. Since this approach is not enough to remove the effect, we proposed the global interpolation with initial solution found by local interpolation step. In our approach using VIS we proposed the method for reduction of constraints thus reducing the computation costs and unwanted oscillations on surfaces.

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