SKULL REGISTRATION WITH MODIFIED INVERSE CHORD LENGTH AND RIGID SUPER-CURVES INCORPORATED WITH SCALING

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ABSTRACT

In this paper, two enhancements are proposed based on 3D Rigid Super-Curves (RSC), which performs rigid registration of two sets of curves by utilizing Super-Curves (SC) where a single B-spline is fitted to both reference and test curves. Firstly, instead of using uniform parameterization in RSC, a modified inverse chord length method is proposed for B-spline parameterisation. It takes the advantage that inverse chord length has much better controllability (i.e. assigning more control points to densely sampled or high variation areas) over other B-spline parameterization methods, and further improves its fitting accuracy by automatically determining the value of the exponential parameter and taking the square-root of the measurements in the original method. Secondly, to further improve the registration accuracy, a uniform scaling factor is incorporated in RSC for the transformation. An iterative algorithm facilitated with some heuristic initial values and stopping criteria is proposed to recover all the transformations alternatively with a fast convergence rate. Experiments on real skull showed the effectiveness of the proposed method.

KEYWORDS

Rigid Super-Curves . B-spline parameterisation . Modified inverse chord length . Controllability . Scaling factor

1. INTRODUCTION

A computer-aided craniofacial clinic system is usually used for craniofacial planning before a surgery is performed. Features are extracted to represent skulls. The system matches the extracted features of the skulls from the same patient at different time and this is known as skull registration. Then surgeons can analyze on the data and make plans before operation is carried out.

Existing methods for skull registration are broadly categorized as points-based, curve-based, and surface-based methods. Point-based methods (Suwardhi et al., 2007) usually represent skulls with multiple sets of point locations. Iterative Closest Points (Besl and McKay, Feb 1992) and Active Shape Model (Hutton et al., 2003) are two examples of point-based methods. For most of these methods the locations of the landmarks are manually placed to establish the correspondence between two skulls (Berar et al., Oct 2004). For surface-based methods, interest surfaces are determined from reference and test images initially. Then minimum distance between these corresponding surfaces is measured. The segmentation involved in these methods is always a high-level task and possibly error-prone (Maintz1 et al., 1996). Curve-based methods do not need to locate landmarks manually, nor do they need to identify known features for registration purposes. Crest lines (Ding and Belaton, Nov 2009)) are usually extracted from the skull to perform registration. B-spline based methods are favored by researchers. Toe and To (Feb 2004) used B-spline (Park and Lee, June. 2007), (Lim, June. 1999), (Sukmarg and Rao, 2002), (Huang and Cohen, Oct. 1996) fitting to perform curve matching. Super-Curves (Xia and Liu, May. 2004) is a B-spline based method that only fits a single B-spline to both reference and test curves. It is intrinsically designed to recover the affine transformation between two 2D images. A rigid Super-Curves method (Liao et al., 11-14, Aug 2009) is proposed based on SC to recover only the rotation and translation between two 3D skull surfaces. However, there are two main problems in the earlier phase of RSC. Firstly, B-spline parameterization is a simple uniform method which does not consider the non-uniform distribution of the sampling points. Secondly, the scaling transformation has not been considered, which may cause errors to registration since the scaling can be an issue if the CT scans are taken at different time and with different machines. Therefore, in this paper we resolve the issue by providing better solution to enhance and improve the accuracy of skull registration.

The paper is structured as follows. Some related work to RSC and B-spline parameterisation are discussed in section 2. In section 3, the modified inverse chord length method for B-spline parameterisation is proposed, followed by an iterative algorithm which incorporating the scaling factor to RSC in section 4. Experimental results and discussion are carried out in section 5. Finally, conclusions and future works are addressed in section 6.

2. RELATED WORK

2.1 Rigid Super-Curves

Rigid Super-Curves was proposed based on SC to recover only rotation and translation between two 3D skull surfaces. The main difference between RSC and SC is its transformation process. When applying B-spline representation for Super-Curves, a rigid transformation (rotation and translation) between the curves is calculated in RSC rather than obtaining the general affine simultaneously in the case of SC. For SC, affine transformation is recovered to match two curves. Transformation includes translation, rotation and horizontal shear:

$$\begin{bmatrix} \tilde{\boldsymbol{x}} \\ \tilde{\boldsymbol{y}} \\ \tilde{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} \mathbf{cos}\,\emptyset & \mathbf{sin}\,\emptyset & 0 \\ -\,\mathbf{sin}\,\emptyset & \mathbf{cos}\,\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{S}_x & 0 & 0 \\ 0 & \boldsymbol{S}_y & 0 \\ 0 & 0 & \boldsymbol{S}_z \end{bmatrix} \begin{bmatrix} \mathbf{1} & \beta & \alpha \\ 0 & \mathbf{1} & \beta \\ \gamma & \mathbf{1} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} + \begin{bmatrix} \boldsymbol{T}_x \\ \boldsymbol{T}_y \\ \boldsymbol{T}_z \end{bmatrix} \qquad (1)$$

In RSC, shearing is not considered because it deals with 3D skulls where shearing rarely happens. Rigid transformation is recovered by independently obtaining the rotation and translation, which preserves the shape of a skull.

2.2 B-spline Parameterisation

B-spline is a function that supports local propagation, smoothness and domain partition. In this paper, it is used to perform curve fitting and registration between two curves.

Let [**P**$_1$, **P**$_2$, …, **P**$_M$] be the M sample points of a curve **P**. It is then divided into L segments and approximated by a linear combination of parametric polynomial basis functions (Xia and Liu, May. 2004).

$$\sum_{i=0}^{M-1} \left\| \boldsymbol{P}_i \; - \; \sum_{j=0}^{L+N-1} \boldsymbol{d}_j \, \boldsymbol{B}_j^N \left(\frac{\boldsymbol{t}_i - \boldsymbol{u}_j}{\boldsymbol{u}_{j+1} - \boldsymbol{u}_j} \right) \right\|^2 \qquad (2)$$

where $\boldsymbol{B}_j^N$ is the B-spline basis function, $\boldsymbol{d}_j$ control points, j = 1, …, L+N-1, N the B-spline degree, $\boldsymbol{t}_i$ is the measurement of sampling points' distance along B-spline curves, $\boldsymbol{u}_j$ the knot vectors, respectively.

Among these parameters the position of the control point is important as it affects both of the accuracy and controllability of the fitted curves. The less fitting error is, the more accurate is the fitted B-Spline. On the other hand, the less sensitive of the spline is to the noise, the better is the controllability. The position of control points along B-spline curves can be determined using the distance between two sampling points. This adjustment is described as B-spline parameterisation, and the parameter is represented as $\mathbf{t}_i$ in the B-spline function as follows.

$$\boldsymbol{t}_i = \boldsymbol{t}_{i-1} + \frac{|\boldsymbol{P}_{i+1} - \boldsymbol{P}_i|^e}{\sum_{i=1}^{M-1} |\boldsymbol{P}_{i+1} - \boldsymbol{P}_i|^e} \qquad (3)$$

where the exponent e = -1,-1/2,0,1/2,1.

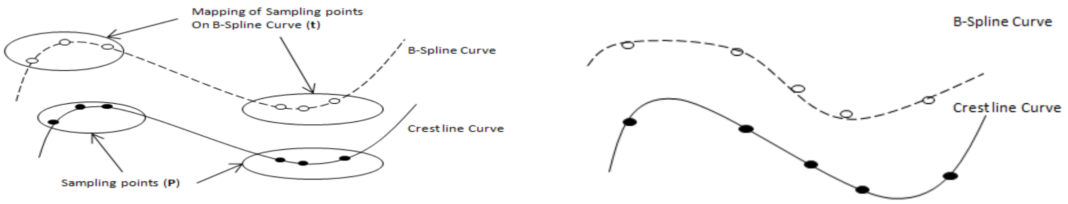
Equation (3) is a general equation to calculate $\mathbf{t}_i$, where different value of e stands for different methods. For example, Equidistant method (Lim, June. 1999) takes e = 0 which only calculates the distance between $\mathbf{t}_i$ using average distance between sampling points. It does not make any proper adjustment between $\mathbf{t}_i$ according to the distance traveled between data points. Chordal or chord length method (Lim, June. 1999), (Park and Lee, June. 2007), (Toe and To, Feb 2004), (Sukmarg and Rao, 2002), (Huang and Cohen, Oct. 1996) takes the value 1 for e. Centripetal parameterisation (Lim, June. 1999) is the modification from chord length where its e is 1/2. Both methods are similar where they are calculating the fraction of the distance traveled between two data points relative to total distance of data points. These methods are performed well on uniformly sampled data because the traveling speed and distance between data points are constant.

We have observed that under the scheme of equation (3), the accuracy and controllability of a B-spline fitting behave contradictorily:

1. To achieve better accuracy, distance of sampling points along B-spline curves must be proportional and correspondent to the sampling points on the crest line curve. $\boldsymbol{t}_{i+1} - \boldsymbol{t}_i \propto \|\boldsymbol{P}_{i+1} - \boldsymbol{P}_i\|$. (Figure 1.a)

2. To achieve better controllability, $\boldsymbol{t}_{i+1} - \boldsymbol{t}_i$ shall be inversely proportional to $\|\boldsymbol{P}_{i+1} - \boldsymbol{P}_i\|$. (Figure 1.b)

It is a paradox to achieve both rules at the same time and it cannot be solved by adding more control points. In addition, too many control points could also over-fit the curve.



(a) Sampling points at feature variation points preserve the shape. (b) Lack of controllability of the sampling points.

Figure 1. Example of curve cases

The chordal and centripetal methods mentioned above only satisfy rule 1, which makes it unsuitable in the cases where curvatures change rapidly, as shown in Fig. 1b, since more control points are needed for highly curved segments. On the other hand, it suffers from non-uniformly sampled data and non-uniformly distributed noise. For non-uniformly sampled data, both the dense and sparse areas appear in one curve, as shown in Fig. 1a. If the chordal is chosen, every curve segment will obtain the same number of control points, and information will be lost in the dense areas as not enough control points are assigned. In real practice, the behaviour of sampled data for skulls is non-uniformly sampled and contains noise. Therefore, the chord length and centripetal methods are not suitable for this research.

Zhaohui Huang et al. (Oct. 1996) used inverse chord length method to overcome the problem. It is the inversion of chord length with e being assigned with a negative value. This makes the traveling time between two points in the densely sampled area inversely proportional to the chord length between them, which slows down the travelling speed between the two points, hence shortens the distance between two control points. Thus it consequently assigns more control points to the densely area as compared to the sparsely area. It has good ability to control shape variation of curves that follows the rule 2. However, the number of high variation or densely sampled areas is required prior to manually assigning an appropriate value to e.

Since the data is non-uniformly sampled, a modified inverse chord length method is proposed which not only maintains controllability of the inverse chord length method, but also improves the accuracy of the original method. A simple equation is proposed to calculate e based on automatically estimating the number of densely sampled or high variation areas. Then, square root is considered as mediation for the distance between points along B-spline curves, which improves the accuracy of the original inverse chord length method.

3. MODIFIED INVERSE CHORD LENGTH METHOD

3.1 Assigning E Automatically

A modified inverse chord length parameterisation is proposed to calculate $\mathbf{t}_i$ as follows.

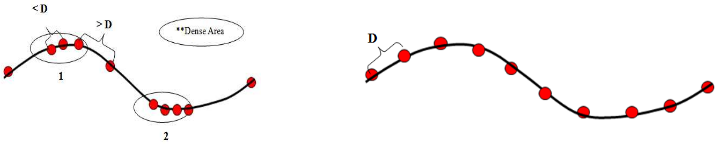
$$\boldsymbol{t}_i = \boldsymbol{t}_{i-1} + \left(\left(\frac{1}{|\boldsymbol{P}_{i+1} - \boldsymbol{P}_i|^{1/e}} \right)^{1/2} . \frac{1}{\sum_{i=0}^{n-1} \left(\frac{1}{|\boldsymbol{P}_{i+1} - \boldsymbol{P}_i|^{1/e}} \right)^{1/2}} \right) \tag{4}$$

Where $\mathbf{t}_1 = 0$, $e > 0$, and $n = \#\{\boldsymbol{P}_i\}$, the total number of sampling points. e value is assigned automatically based on number of high variation or densely sampled areas. Based on Zhaohui Huang and Fernand S. Cohen (Oct. 1996), where it was experimentally found that 4 gave the best results for the registration in their case with non-uniformly sampled data. They showed empirically that the smaller e, the slower the speed between two consecutive data points and more control points in the dense area.

In order to assign e automatically, the following method is proposed. Firstly, average distance between two sampling points is calculated, as shown below:

$$\boldsymbol{D} = \frac{\sum_{i=1}^{M} |\boldsymbol{P}_{i+1} - \boldsymbol{P}_i|}{\boldsymbol{M}} \tag{5}$$

where **D** is the average distance between sampling points, $\mathbf{M} = \#\{\boldsymbol{P}_i\}$, $\boldsymbol{P}_i$ is sampling points. It is then used to compare with the distance between each two successive sampling points to find out the total dense or high variation areas. Fig. 2 demonstrates the way to determine the dense or high variation areas.



(a) **(b)**

Figure 2. Searching of dense areas

In Figure. 2a, notices that there are two dense areas where more sampling point are located. These sampling points are not distributed evenly and the distance between these sampling points are not equal. In dense areas, the distance between two sampling points is smaller. By comparing average distance and its distance between two successive sampling points, it can determine whether sampling points in that area is dense or sparse. The average distance between each two successive sampling points is calculated based on equation (5). Fig. 2b shows the pattern when the distance between sampling points are in equilibrium. The calculation for that area continues until distance between two sampling points is larger than the average, as shown above. A marker is then placed in that area. For Fig. 2a, there will be two markers denoting two dense areas. Then the number of markers is used to calculate e as follows:

$$\mathbf{e} = 4 - \ln(N - 1.5) \tag{6}$$

where N is the number of total dense areas found. Fig. 3 shows the variation of e against the number of the dense areas. When the number of areas increases, e becomes smaller. The smaller the e, the more control points assigned to the dense or high variation areas. The constant values - 1.5 and 4 in the equation also affects the value of e. If the constant is large, the value of e increases. They are adjusted according to the experiment. A small sample experiment using real data has been carried out to observe the value of e based on various constants. 5 curves are adopted to observe e from set of curves in real data. From the Table 1, notice that, as the value of the constant increased, the value of e also increased. However, when the number of densely sampled or high variation areas is high, e should be smaller. Hence, the constant must not be large because it will cause e to increase. On the other hand, when the value of the constant is small, e is too small and becomes negative, which is invalid._Hence, 4 is the suitable value of the constant for the equation. For example, in curve No. 3, the number of dense areas found is 9 which is considered as a large number. Hence, e must be small. By observing Table 1, 6 has provided higher e than 4, and 2 has provided negative value which is invalid. Since small e value is required, 4 is considered as the suitable constant for the equation.

Table 1. Observations of e based on different constants

	Constant N	2	4	6
1	3	1.5945	3.5945	5.5945
2	1	6.0000	6.0000	6.0000
3	9	-0.0149	1.9851	3.9851
4	1	6.0000	6.0000	6.0000
5	6	0.4959	2.4959	4.4959

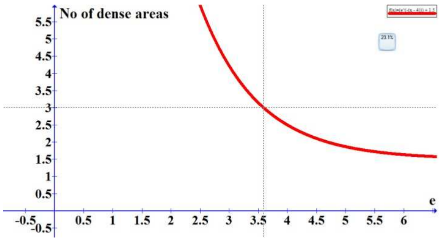


Figure 3. Example for finding e

In Fig. 3, it shows that if there are 3 dense areas, e will be 3.59.

The square root is applied to the modified inverse chord length to increase the accuracy and maintain the number of control points in dense or high variation areas. By maintaining the number of control points in these areas, it will obtain an accurate fitting between sampled curves and B-spline curves.

4. RIGID SUPER-CURVES BY INCORPORATING SCALING TRANSFORMATION

Scaling is incorporated in RSC for recovering the transformation between two curves and it is estimated after obtaining the rotation and translation. The three types of transformation (rotation, translation and scaling) are dealt with separately as only in this way the property of the corresponding transformation can be controlled and maintained. The method is shown as follows.

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}\mathbf{cos}\,\boldsymbol{\phi} & \mathbf{sin}\,\boldsymbol{\phi} & 0\\ -\,\mathbf{sin}\,\boldsymbol{\phi} & \mathbf{cos}\,\boldsymbol{\phi} & 0\\ 0 & 0 & \mathbf{1}\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \begin{bmatrix}\boldsymbol{T}_x\\ \boldsymbol{T}_y\\ \boldsymbol{T}_z\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}\vec{x}\\ \vec{y}\\ \vec{z}\end{bmatrix} = \begin{bmatrix}\boldsymbol{S}_x & 0 & 0\\ 0 & \boldsymbol{S}_y & 0\\ 0 & 0 & \boldsymbol{S}_z\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} \tag{8}$$

Equation (7) is considered in RSC, where the rotation and translation are calculated using Singular Value Decomposition technique (Liao et al., 11-14, Aug 2009). Here we use equation (8) to incorporate the scaling transformation.

To register the reference and the test skulls, we have:

$$\mathbf{min}\ \|\text{reference} - (\mathbf{R}\text{test} + \mathbf{T})\|^{\mathbf{2}} \tag{9}$$

$$\mathbf{min}\ \|\text{reference} - (\mathbf{S}.\mathbf{R}\,\text{test} + \mathbf{T})\|^{\mathbf{2}} \tag{10}$$

Where **S** is scale, **R** is rotation, **T** is translation, reference is reference skull and test is test skull.

Equation (9) and equation (10) are used to calculate matching/registration errors between reference and test images. Control points from every B-spline curves are used to calculate the registration errors. Equation (9) is used to calculate registration errors without scaling. For equation (10), scaling is considered. The purpose to create these two equations is to compare the registration before and after scaling.

Fig. 4 shows the flow of the proposed iterative algorithm. When the position and size of the test skull has fitted with the reference skull, their values become constant and it is considered as having achieved good matching. When any of the operations (rotation, translation and scaling) has achieved good matching, the iteration process stops. The stopping criteria are proposed as follows in this research:

1. Rotation and translation become void

The value of the rotation and translation change in each iteration loop and ideally their values are void where the rotation matrix becomes the identity matrix and the translation becomes the zero vector when reference and test skulls match with each other. Hence, iteration should stop.

2. No changes on registration errors

When the value of registration errors between two successive iterations is lower than a predefined threshold, the algorithm finds an optimal solution and stops.

3. Stopping criteria for scaling

Stopping criteria for scaling is different from rotation and translation. An estimated ratio is calculated from the initial reference and test skulls to get the initial size of the difference between the two skulls before scaling is applied. Given, reference curve as $\mathbf{P}_i$, i = 1, …,M Test curve as $\mathbf{Q}_i$, i = 1, …, N

$$Estimated\ Ratio = \frac{\sum_{i=1}^{M-1}\|\boldsymbol{P}_i\|}{\sum_{i=1}^{N-1}\|\boldsymbol{Q}_i\|} \tag{11}$$

	Constant N	2	4	6
1	3	1.5945	3.5945	5.5945
2	1	6.0000	6.0000	6.0000
3	9	-0.0149	1.9851	3.9851
4	1	6.0000	6.0000	6.0000
5	6	0.4959	2.4959	4.4959

A new set of test skulls (scaled) will be obtained after scaling factor is applied. A new estimated ratio is calculated with the new set of test skulls using equation (11). It is then compared with the estimated ratio calculated earlier. The new estimated ratio must be bounded within initial ratio and approximate 1.0, meaning the size of test skull is almost identical to the reference skull. It is to verify whether the size of the new test skull has diverged from the reference skull. If it begins to diverge from the boundary, the scaling process stops. For example, if the initial estimated ratio is 0.5, it means the size of test skull is twice as much as the one of the reference skull. The new ratio is expected to be between 0.5 and 1.0 during each iteration. When the estimated ratio is higher than 1.0, it means that the test skull has become smaller than the reference skull and scaling must be stopped. Besides, an initial guess shown in equation (12) for scaling is determined during first iteration to accelerate the registration process. This guess comes from the ratio of the closest reference and test curves. The closest curves are the pair of curves that corresponds to each other among the set of curves in reference and test skulls. Given, reference point as **P**$_i$, i = 1, …, M Test point as **Q**$_i$, i = 1, …, N

$$Initial\ guess = \frac{\sum_{i=1}^{M-1} |\boldsymbol{P}_i - \boldsymbol{P}_{i-1}|}{\sum_{i=1}^{N-1} |\boldsymbol{Q}_i - \boldsymbol{Q}_{i-1}|} \qquad (12)$$

$$\boldsymbol{S} = argmin \left\| \mathbb{V}' - \boldsymbol{S}.\mathbb{V} \right\|^2 \qquad (13)$$

Equation (13) is to find scaling value where V' represent a set of reference and V represent a set test data set using vector form. The size for each of the vector is n X 3. Let

$$f(\boldsymbol{S}) = \left\| \mathbb{V}' - \boldsymbol{S}.\mathbb{V} \right\|^2 \qquad (14)$$

Since f(**S**) is quadratic and convex as shown in the following equation, it must have a minimum point.

$$f(\boldsymbol{S}) = (\mathbb{V}' - \boldsymbol{S}.\mathbb{V})^T (\mathbb{V}' - \boldsymbol{S}.\mathbb{V})$$

$$= \left(\mathbb{V}_1' - \boldsymbol{S}.\mathbb{V}_1, \ldots\ldots, \mathbb{V}_{3n}' - \boldsymbol{S}.\mathbb{V}_{3n}\right)^T (\mathbb{V}_1' - \boldsymbol{S}.\mathbb{V}_1, \ldots\ldots, \mathbb{V}_{3n}' - \boldsymbol{S}.\mathbb{V}_{3n})$$

$$= \sum_{i=1}^{3n} \left(\mathbb{V}_i' - \boldsymbol{S}.\mathbb{V}_i \right)^2 \qquad (15)$$

To find the minimum value for f(**S**), it just needs to solve the equation $\frac{df}{ds}$ = 0, i.e.

$$\frac{df}{ds} = \sum_{i=1}^{3n} 2(\mathbb{V}_i' - \boldsymbol{S}.\mathbb{V}_i).(-\mathbb{V}_i) = 0 \qquad (16)$$

which results in

$$\boldsymbol{S} = \frac{\sum_{i=1}^{3n} \mathbb{V}_i \mathbb{V}_i'}{\sum_{i=1}^{3n} \mathbb{V}_i^2} \qquad (17)$$

Equation (10) can be written as $\mathbf{min}\|(reference - \boldsymbol{T}) - \boldsymbol{S}.(\boldsymbol{R}\ test)\|^2$ which is similar to equation (14) – $\min\|\mathbb{V}' - \boldsymbol{S}.\mathbb{V}\|^2$. Let $\mathbb{V}' = reference - \boldsymbol{T}$ and $\mathbb{V} = \boldsymbol{R}.test$. Obtain:

$$\boldsymbol{S} = \frac{\sum_{i=1}^{3n} (\boldsymbol{R}.\boldsymbol{d}_i)(\boldsymbol{d}_i')}{\sum_{i=1}^{3n} (\boldsymbol{R}.\boldsymbol{d}_i)} \qquad (18)$$

where **S** is the scaling factor and **d'** is control points for reference curves and **d** is control points for test curves.

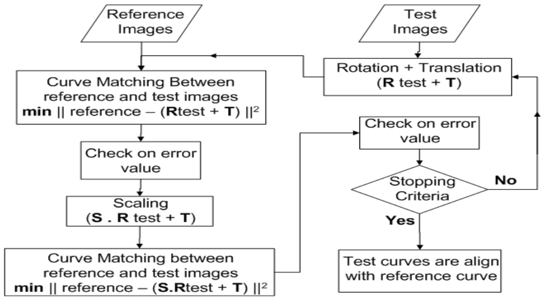


Figure 4. Iteration process to gain good matching in transformation

5. RESULT AND DISCUSSION

A set of real (skull) data is used in the experiments. Crest lines are extracted from the real CT skull. To test and compare modified inverse chord length with inverse chord length, experiments are carried out for each technique without rotation, scaling and translation. To test on the scaling method, test skull is adjusted using different scale factors to create different size of reference and test skull. The scale factors are set as 0.25 and 2.0 to test on the robustness of the scaling. The maximum number of iterations is set as 100.

5.1 B-spline Fitting

The evaluation for B-spline parameterisation is based on equation (19), equation (20) and equation (21) which is described as B-spline fitting error. The data set that used is the control points that generated from B-spline basis function.

$$\mathrm{B\_Err1} = \sum_{i=1}^{M} \|\mathrm{H}_1\mathbf{d}_x - [\mathrm{P}_x]\|^2 + \left\|\mathrm{H}_1\mathbf{d}_y - [\mathrm{P}_y]\right\|^2 + \|\mathrm{H}_1\mathbf{d}_z - [\mathrm{P}_z]\|^2 \quad (19)$$

$$\mathrm{B\_Err2} = \sum_{i=1}^{N} \left\|\mathrm{H}_2\tilde{\mathbf{d}}_x - [\tilde{\mathrm{P}}_x]\right\|^2 + \left\|\mathrm{H}_2\tilde{\mathbf{d}}_y - [\tilde{\mathrm{P}}_y]\right\|^2 + \left\|\mathrm{H}_2\tilde{\mathbf{d}}_z - [\tilde{\mathrm{P}}_z]\right\|^2 \quad (20)$$

where B_Err1 is B-spline fitting error for the reference, B_Err2 is B-spline fitting error for the test skull and H is determined by B-spline basis function (May. 2004), (Liao et al., 11-14, Aug 2009). The average error value is defined as

$$\mathrm{B\_Err} = \frac{\mathrm{B\_Err1} + \mathrm{B\_Err2}}{\mathrm{M+N}} \quad (21)$$

Comparison is made between the inverse chord length and the proposed modified inverse chord length method.

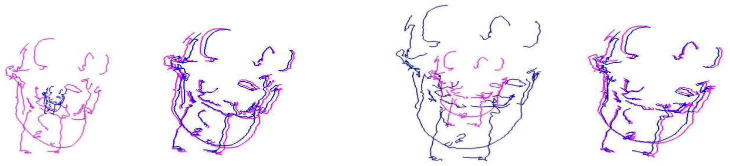
Table 2. B-spline parameterisation between inverse chord length and modified inverse chord length for real data

B-spline Parameterisation Techniques	B-spline Fitting Errors
Inverse Chord Length	1.81636
Modified Inverse Chord Length	1.66147

Notice that the proposed modified inverse chord length method has achieved lower fitting error compare to the inverse chord length method. It has maintained the controllability and also increased the accuracy of B-spline fitting. The accuracy increased because of the adjustment of e for each of the curves. e is assigned based on the characteristics of the curves as shown in section 3.1. Besides, the square root operation has become the mediation for the distance between the points on B-spline curves.

5.2 Transformation

The experiment is to test whether the scaling can scale the test skull to a bigger/smaller size to match the reference skull if the test skull is bigger/smaller than the reference skull as shown in Fig. 5. For example, in Fig. 5a, in the before case, the big skull is the reference and the small skull is the test. The registration error with scaling is calculated based on equation (10).



(a) Before and After (test skull < reference skull) (b) Before and After (reference skull < test skull)

Figure 5. Before and After scaling for transformation

From Fig. 5a, the size of the test skull is smaller than the reference skull before the scaling is applied. They do not match with each other. The registration error is large as it is shown in Table 3a. In Table 3a, after the 2nd loop, the registration error remains the same compared to the one in the 1st loop. Hence, it stops.

Table 3. Registration error of skulls with scale factor 0.25 with scaling

	(a) Skulls with Scale factor 0.25 without Scaling	(b) Skulls with Scale factor 0.25 with Scaling
Loop	Registration Error	Registration Error
1	2109.96	2109.96
	-	14900.1
2	2109.96	244.879
	-	244.576
3	-	244.88
	-	244.037
4	-	244.88
	-	244.037

From Table 3b, the registration error value is lower compared with the one in Table 3a when the test skull matched with the reference skull. There is only 4 iterations. In every loop, we compared the registration error before and after the scaling using equation (9) and equation (10) respectively. The first row is before scaling and the second row is after scaling as shown in Table 3b. It stops when the error remains unchanged as compared to the previous loop. The initial guess and stopping criteria proposed in section 4 have accelerated the registration process. The Fig. 5c and 5d is the case where the test curves are scaled to 2.0 times as large as the reference curves. The test skull has successfully matched with the reference skull as shown in Fig. 5d. In summary, the scaling and iteration techniques in the proposed algorithm have provided promising results.

6. CONCLUSIONS AND FUTURE WORKS

In this paper, the previous problems - B-spline parameterisation and scaling transformation in RSC are addressed for skull registration. The proposed modified inverse chord length has increased the accuracy of the B-spline fitting compared to the original inverse chord length method, by automatically assigning exponential parameter value and adding square root to the measurement in the original method. For transformation, a uniform scaling factor is incorporated in RSC and an iterative algorithm is proposed to further improve the registration. The initial guess and stopping criteria are estimated with the heuristics provided in the data, to accelerate the matching process between the reference and test skulls. For future work, the accuracy or controllability can be further improved by adjusting other parameter in B-spline equation. The suggested parameters can be knot vectors. Knot vectors will determine the areas where control points should have been placed. The equation to determine the value of the exponential parameter in the proposed modified inverse chord length method can also be improved. Applying scaling in different skulls shall be tested in the future. Also, other methods can be utilised to find the initial guess for the scaling factor.

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