ESP: EUCLIDEAN SPHERES PACKING IMPLEMENTATION FOR VOLUME DATA USING SLICER3D

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ABSTRACT

The Gamma Knife radiosurgery treatment involves geometrically mapping for covering the tumor area in three dimensions so that the gamma rays can provide precise treatment to the affected area. The entire process has to be accurate in order to prevent the exposure of the gamma rays to the neighboring healthy tissues. When the tumor shape is irregular, it is difficult to ensure the coverage of the tumor region. We present, in this paper, a new approach to fit spheres in a bounded region based on maximum distance. Our proposed method for automating the gamma knife surgery planning is by using a geometrically-based Euclidean sphere packing optimization method. We implement our algorithm in Slicer3D using Python libraries. Experiments show that our algorithm produces reasonable dense packing of different irregular shapes.

KEYWORDS

Euclidean Distance, Slicer3D, Sphere Packing, Gamma Knife, Stereotactic Radiosurgery

1. INTRODUCTION

Spheres packing is a venerable topic in mathematics. Researchers have been interested in packing spheres into regions since the time of mathematician and astronomer Johannes Kepler (Szpiro, 2003). Different sphere packing problems have been studied depending on the shape, type, and position of objects. Our study is motivated by medical applications in radiosurgery. Gamma knife is one type of the radiosurgery that is a minimally invasive procedure uses radiation to kill tumor tissue inside the human brain. A main geometric problem in Gamma knife treatment planning is to fit multiple radiation shots into a 3D irregular shaped tumor. The Gamma Knife works by a process named Stereotactic Radiosurgery (SRS) where each shot is consisting of gamma-ray beams coming from different sectors with different collimation that can be represented as spheres of diameters (Ferris & Shepard, 2000).

Performing shape approximations are required to ensure that the dosage does not go outside the geometric coverage area to preserve as many normal tissues as possible. The general approach to generate spheres or approximate shape in Gamma knife is by placing random spheres or by using skeleton (Wu, Q. Jackie & Bourland, 1999; Wu, Qingrong J., Bourland, & Robb, 1996). These approaches can significantly change the behavior in Gamma Knife surgery. The reason for that is because the skeleton algorithms check only the end points and cross points of the skeleton at each iteration to place a sphere without caring about the other object's points. As a complete algorithm for arbitrary shapes that uses the only *minimum* number of spheres based on maximum distance is NP-Complete, taking exponential time in the worst case. The radiation planes are only targeting tumor tissues that need to be eliminated and destroy their DNA, so it limits the intensity so that tumor tissues will be affected more than the normal tissues. Margins around the tumor volume are strongly depend on the clinical assessment of risk and extent of spread, such as in the case of benign and malignant tumors. In addition, there are two different types of radiotherapy, frame-based and frameless system. In the frame-based SRS system, a fixed immobilization frame is used to position the patient's head to the correct coordinates for each shot, so the dose is delivered to tumor using MRI without margins. In the frameless, margins are necessary around tumors in case of target motion occurs. Adding many margins could expose more normal brain tissue to radiation and increase the risk. In this paper, we only focus on the frame-based SRS that target the tissues that are to be eliminated without margins, which could be added later by the radiologists. Moreover, certain sizes of shots (spheres) could be hard to get or unavailable because of physical constraints. In our case, we

choose the placement covering the largest number of voxels inside the tumor no matter what is the size of the spheres. This is because our goal is to design the treatment planning system that optimize the target coverage, not proposing a design suitable for routine clinical practice. Optimizing conformity, where finding number of beams needed with their weights and angles, are not our focus, which in other word are determined by a dosimetrist.

Our *approximate* algorithm for sphere packing provides the possibility to group several spheres together to cover complex structures based on the maximum distances to the object boundary. Distance transformation is not new but when used in our algorithm it creates a way to generate spheres iteratively, which is different than the existing methods. We place the spheres such that they do not overlap and do not intersect with healthy tissues surrounding the tumor since overlapping may cause an overdose. The proposed algorithm offers a novel method to create multi-spheres for the given arbitrary shapes for more than 90% of the tumor coverage and terminates in a reasonable time based on the number of spheres and coverage desired.

2. THE CREATION OF ESP

The spheres are placed into the tumor target one-by-one, starting with the largest sphere based on the maximum Euclidean distance to tumor boundary. Our main goal is that the treated volume (covered by the spheres) matches the tumor shape by placing spheres that cover as many tumor pixels as possible to ensure the coverage of tumor tissues. The total sphere coverage density and radii can be adjusted to obtain different purposes such as in Gamma Knife surgery. Three objectives need to be fulfilled for sufficient approximation: using the minimum number of spheres is required for saving computation time, the applicability of the algorithm for different arbitrary shapes, and the number of spheres must be enough to accomplish an adequate surface representation. Our ESP algorithm consists of:

A. Find the Segmented Region:

Medical imaging is given as a 3D surface representation by using medical imaging techniques. The standard file format in this context is DICOM that represent the images as a complete 2D slices. We access the segmented tumor image (label map) as an array. The label map data is composed of a 3-D array of data, which specify the x , y , and z coordinates for each data point while the background value=0.

B. Get Range of Region:

It is necessary to mark the region boundary since that characterizes the volume surface. We get the boundaries of our region of interest (tumor) by excluding the object's pixels that are outside of our region of interest. To do that, we first use a bounding box by extracting both the minimum (x_{mn} , y_{mn} , z_{mn}) and the maximum (x_{mx} , y_{mx} , z_{mx}) non-zero cubic values of the corresponding coordinate (Figure 1) for faster geometrical operations and reducing the calculation time. Second, we delete all zero's pixels, which are background pixels.

C. Apply Distance Transform:

Calculate the 3D exact Euclidean distance transformation of each pixel as explained in our paper in $O(n)$ time (Alhazmi & Semwal, 2018).

Figure 1. The segmented region that shows the tumor region before and after the bounding box

Figure 2. Find the coordinates of points within a sphere bounded by shape. (a) grid points. (b) mask of value=0. (c) The Boolean distance of all points from the center (d) The inner part of sphere will have distance < 1 and border = 1

D. Record the center and radius of sphere.

Determine the biggest sphere by using the maximum distance value as a radius “ r ” and its location as center “ c ”. To find the exact location of the sphere’s center, the distance between the center of the sphere and the center of each bounding cell is computed. So, the minimum coordinate values along each axis (xmn , ymn , zmn) is added to the center value, as in: $center = [c[0][0] + xmn, c[1][0] + ymn, c[2][0] + zmn]$ (1)

Then, save these sphere’s information into two lists: one for radiuses “ $r_list[]$ ” and the other one for the centers “ $c_list[]$ ”.

E. Getting further spheres:

Generate a center-based grid, which is a grid constructed around the sphere’s center, stretched by the shape of the segmented region (Figure 2 (a)). In the grid, each voxel represents a volume of fixed size. Then, we use a mask of the same size with zero values (Figure 2 (b)) to calculate the Boolean distance of each point from the sphere’s center scaled by the radius (Figure 2 (c)). Positive voxels (sphere’s voxels) are labeled by the distance of these voxels while surrounding tissue voxels are represented by zero (Figure 2 (d)). The area of the sphere by definition is:

$$(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = r^2 \quad (2)$$

Where (x,y,z) is a point, (x_c,y_c,z_c) is the center, and r is the radius. So:

$$((x-x_c)/r)^2 + ((y-y_c)/r)^2 + ((z-z_c)/r)^2 = 1 \quad (3)$$

Points inside the sphere will have the lowest values (below 1), and points on the border will have the highest value (equal to 1) (Figure 2 (d)). After that, we need to exclude the first sphere’s pixels from the segmented region to start placing the next sphere, so we subtract one from the previous mask in (Figure 2 (c)) as in (Figure 3). The final result is as in (Figure 4).

Figure 3. (a) mask of the sphere’s shape. (b) converting (a) to float type. (c) the result after subtracting 1 from (b) with sphere’s points=0

Figure 4. (a) sphere’s shape =0, multiply by (b) segmented region. (c) the result of the segment region without the pixels/voxels of the first sphere

F. Calculate the approximation error:

The ratio between current non-zero volumes and original non-zero volumes. This ratio is used to find when our algorithm terminates based on the number of spheres and coverage (*accuracy rate*) desired. If the: $approximation\ error < (accuracy\ rate * original\ non-zero\ volume)$, then draw all spheres, otherwise, go to step C. The complete algorithm of approximation is shown as pseudo code, as follow:

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1. Input volume as nrrd file
2. SegmentedRegionOrigin= Volume (nrrd)
3. Create Bounding Box
4. SegmentedRegion= Bounded Regions with margins
5. OriginalVolume= SegmentedRegion (true)
6. Distance= EuclideanDistance (SegmentedRegion)
7. Max= maximum distance
8. Sphereradius= max
9. Spherecenter= location (max) // center inside bounding box
10. Spherecenter= (Spherecenter + nrrd) // center inside nrrd volume
11. Spheredist= (distance, radius, center)
12. SegmentedRegion= (1- Spheredist)
13. While (pixels not covered > pixels in desired coverage) do
    Repeat 6, 7, 8, 9, 10, 11, 12
14. Draw MultiSpheres()
15. Update volume (outputVolume)

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3. RESULT AND DISCUSSION

We implement our algorithm as a new module in Slicer3D (Fedorov, 2012) (Figure 5). It is an open-source medical visualization application that builds on top of different libraries and provides a variety of tools or as in Slicer3D called modules. Our Experiments on MacBook 10.12.6 is based on the BRATS dataset (Menze, Bjoern H and Jakab, 2015) that contains large brain tumor MRI scans with ground truth. The MRI scans and all the ground truth labels are manually revised and delineated by experts board-certified neuroradiologists. In our study, we did our experiments on 3 datasets, each with 10 patients, with different tumor size, intensity, and shape. The acceptance rate of the coverage that been used in medical applications is above 90% (Wang, 1999), the remaining voxels can still be covered by the beams defined by those spheres (multiple beams create a dose like a cloud that cover the gaps in between spheres). The resulting dose distribution shows that gaps between shot boundaries (spheres) are covered in most cases with adequate dose, when using 50% isodose line (Landberg et al., 2016). In this paper, due to the fact that overlapping is not allowed, we believe that our sphere packing algorithm usually requires much higher number of spheres to reach the desired density instead of using overlapping spheres.

Figure 5. Sphere packing module GUI in Slicer3D

Figure 6. ESP 3D result in Slicer3D with 50%, 60%, 70% of packing density

The creation of multiple spheres or isocenters is always a compromise between the total number of spheres and accuracy (packing density) (Figure 6). Increasing the approximation accuracy implies to increase the number of spheres (Figure 7). In other words, the approximation accuracy can be assumed as a score of similarity to the original shape. More spheres give better shape approximation, and an infinite number of spheres will give an identical shape approximation. Moreover, the minimum spheres' radii decrease when more spheres are added because adding more spheres leave a small free area that needs to be covered to give more dense packing. Therefore, a number of small spheres (1 pixel) are distributed around the 3D shape especially around its boundary.

Depending on the total number of pixels/voxels in the shape, the number of the filled spheres and radii may vary significantly. The maximum spheres radii increase with the increase of tumor size. Increasing the tumor size also leads to increase the number of spheres with respect to the shape complexity. Complex tumor shape affects the result such as increasing the number of spheres. Regular tumor shape, on the other hand, decreases that number of spheres because large spheres can cover a large amount of the tumor's pixels.

The computation time of the sphere packing algorithm depends on different criteria such as the total number of pixels/voxels in the shape and accuracy (density). From (Figure 8), we can see that time increases rapidly when increasing the dense packing from 50% to 70%. Our algorithm sometimes takes long time in case of high dense packing such as in 90%. This long time is because of the need of computing the 3D arrangement of $O(n)$ complicated surfaces while nontrivial numerical manipulations are needed.

4. CONCLUSIONS AND FUTURE WORKS

Gamma knife currently produces spheres of three different radii (4mm, 8mm, and 16 mm). In our method, we present a new approach to fit spheres in a bounded region based on maximum distance. Due to the shape irregularity of the 3D object (tumor), approaches such as using skeleton and grassfire may fragment the object into small disconnected regions that remain eventually with no further packing. The ESP is a novel technique to approximate a shape to be used in Radiosurgery. Experiments show that our algorithm produce reasonable

dense packing of different irregular shapes. Using the grid of shape ensure that the shape is appropriately filled. The coverage percentage of the shape grid points is adjustable to reach a variant density with a sufficient number of spheres for any arbitrary shape. The radii of all spheres are computed as the maximum Euclidean distance to the shape boundary.

For the future works, using machine learning and optimization algorithms in the future could reduce the number of spheres, such as using clustering algorithms to cluster multiple spheres into one sphere such that those sphere's centers are very close to each other. There is another criterion that would give better results such as controlling spheres allocation and spacing between spheres at fixed distances. In addition, using a tight margin around the shape could also be considered especially in the case of very small shapes to ensure accurate coverage. Moreover, further investigations into Gamma Knife treatment planning, doses size, and weights need to be pursued in the future as well.

Figure 7. Increasing the number of spheres when more dense coverage is required

Figure 8. Execution time increases for dense spheres packing

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